Title: How to obtain the Tutte polynomials of graphs from representation-theoretic methods?

Abstract: Let G be a graph, and let T_G(x,y) denote its Tutte polynomial, one of the most fundamental invariants associated with G. The Tutte polynomial encodes deep combinatorial information, including spanning trees, graph colorings, flows, and connectivity properties.

In 2015, Viswanath and I proved that the chromatic polynomial of G arises from the root multiplicities of the Kac–Moody algebras whose quasi Dynkin diagrams are G. Since the chromatic polynomial can be expressed (up to sign) as qT_G(1-q,0), this establishes a connection between the Tutte polynomial and the root multiplicities of Kac–Moody algebras along the x-direction.

On the other hand, a recent result of Tarig Abdelgadir, Anton Mellit, and Fernando Rodriguez-Villegas (Proc. Roy. Soc. Edinburgh Sect. A, 2022) shows that the specialization T_G(1,q), corresponding to the y-direction, appears as a special case of the Kac polynomial, which counts absolutely indecomposable finite-dimensional representations of a quiver Q with underlying graph G. This provides a complementary link between the Tutte polynomial and the representation theory of quivers in the y-direction.

Taken together, these results strongly suggest the existence of a unified representation-theoretic framework for the entire Tutte polynomial. In this talk, we will see how T_G(x,y) can be realized as the bigraded Hilbert–Poincaré series of the cohomological Hall algebra of a quiver Q with underlying graph G. The two gradings correspond respectively to the PBW decomposition of the quantum group and the cohomological grading on moduli spaces of quiver representations. This is work in progress.